A Novel Algorithm of Matrix Computation for Lower Approximation Reduction in IOIS

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Abstract—In this article, a new method for lower approximation reduction is introduced in inconsistent ordered information systems(IOIS). The dominance matrix and lower approximation decision matrix are proposed in information systems based on dominance relations. Furthermore, the algorithm of lower approximation reduction is obtained, from which we can provide approach to lower approximation reduction operated simply in inconsistent systems based on dominance relations. Finally, an example illustrates the validity of this method, and shows the method is excellent to a complicated information system.

Index Terms—Dominance relation; Information system; Rough set; lower approximation reduction

I. INTRODUCTION

The rough set theory, proposed by Pawlak in the early 1980s[1], is an extension of the classical set theory for modeling uncertainty or imprecision information. The research has recently roused great interest in the theoretical and application fronts, such as machine learning, pattern recognition, data analysis, and so on.

Attributes reduction is one of the hot research topics of rough set theory. Much study on this area had been reported and many useful results were obtained until now[2-7]. However, most work was based on consistent information systems, and the main methodology has been developed under equivalence relations(indiscernibility relations). In practise, most of information systems are not only inconsistent, but also based on dominance relations because of various factors. The ordering of properties of attributes plays a crucial role in those systems. For this reason, Greco, Matarazzo, and Slowinski[8-13] proposed an extension rough sets theory, called the dominance-based rough sets approach(DRSA) to take into account the ordering properties of attributes. This innovation is mainly based on substitution of the indiscernibility relation by a dominance relation. In DRSA, where condition attributes and classes are preference ordered. And many studies have been made in DRSA[14-20]. But simpler results of knowledge reductions are very poor in inconsistent ordered information systems until now.

In this paper, the method operated simply for lower approximation reduction is introduced in inconsisten is obtained, from which we can provide new approach to knowledge reductions in inconsistent systems based on dominance relations. Finally,

an example illustrates the validity of this method, and shows the method is excellent to a complicated information system.

II. ROUGH SETS AND ORDERED INFORMATION SYSTEMS

The following recalls necessary concepts and preliminaries required in the sequel of our work. Detailed description of the theory can be found in [4,5].

An information system with decisions is an ordered quadruple $\mathcal{I} = (U, A \cup D, F, G)$, where

$U = \{x_1, x_2, \cdots, x_n\}$ is a non-empty finite set of objects;

$A \cup D$ is a non-empty finite attributes set;

$A = \{a_1, a_2, \cdots, a_p\}$ denotes the set of condition attributes;

$D = \{d_1, d_2, \cdots, d_q\}$ denotes the set of decision attributes, and $A \cap D = \phi$;

$F = \{f_k | U \to V_k, k \le p\}$, $f_k(x)$ is the value of a_k on $x \in U$, V_k is the domain of $a_k, a_k \in A$;

$G = \{g_{k'} | U \to V_{k'}, k' \le q\}$, $g_{k'}(x)$ is the value of $d_{k'}$ on $x \in U$, $V_{k'}$ is the domain of $d_{k'}, d_{k'} \in D$.

In an information systems, if the domain of a attribute is ordered according to a decreasing or increasing preference, then the attribute is a criterion.

Definition 2.1(See [4]) An information system is called an ordered information system(OIS) if all condition attributes are criterions.

Assumed that the domain of a criterion $a \in A$ is complete pre-ordered by an outranking relation $\succcurlyeq_a$, then $x \succcurlyeq_a y$ means that x is at least as good as y with respect to criterion a. And we can say that x dominates y. In the following, without any loss of generality, we consider condition and decision criterions having a numerical domain, that is, $V_a \subseteq \mathcal{R}(\mathcal{R}$ denotes the set of real numbers).

We define $x \succcurlyeq y$ by $f(x, a) \ge f(y, a)$ according to increasing preference, where $a \in A$ and $x, y \in U$. For a subset of attributes $B \subseteq A$, $x \succcurlyeq_B y$ means that $x \succcurlyeq_a y$ for any $a \in B$. That is to say x dominates y with respect to all attributes in B. Furthermore, we denote $x \succcurlyeq_B y$ by $x R_B^{\geqq} y$. In general, we indicate a ordered information systems with decision by $\mathcal{I}^{\succcurlyeq} = (U, A \cup D, F, G)$. Thus the following definition can be obtained.

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an ordered information system with decisions, for $B \subseteq A$, denote

$$R_B^{\succeq} = \{(x_i, x_j) \in U \times U | f_l(x_i) \geq f_l(x_j), \forall a_l \in B\};$$

$$R_D^{\succeq} = \{(x_i, x_j) \in U \times U | g_m(x_i) \geq g_m(x_j), \forall d_m \in D\}.$$

$R_B^{\succeq}$ and $R_D^{\succeq}$ are called dominance relations of information system $\mathcal{I}^{\succeq}$.

If we denote

$$
\begin{aligned}
[x_i]_B^{\succeq} &= \{x_j \in U | (x_j, x_i) \in R_B^{\succeq}\} \\
&= \{x_j \in U | f_l(x_j) \geq f_l(x_i), \forall a_l \in B\}; \\
[x_i]_D^{\succeq} &= \{x_j \in U | (x_j, x_i) \in R_D^{\succeq}\} \\
&= \{x_j \in U | g_m(x_j) \geq g_m(x_i), \forall d_m \in D\},
\end{aligned}
$$

then the following properties of a dominance relation are trivial.

Proposition 2.1(See [4])   Let $R_A^{\succeq}$ be a dominance relation. The following hold.

(1) $R_A^{\succeq}$ is reflexive,transitive, but not symmetric, so it is not a equivalence relation.

(2)   If $B \subseteq A$, then $R_A^{\succeq} \subseteq R_B^{\succeq}$.

(3)   If $B \subseteq A$, then $[x_i]_A^{\succeq} \subseteq [x_i]_B^{\succeq}$.

(4)   If $x_j \in [x_i]_A^{\succeq}$, then $[x_j]_A^{\succeq} \subseteq [x_i]_A^{\succeq}$, and $[x_i]_A^{\succeq} = \cup\{[x_j]_A^{\succeq} | x_j \in [x_i]_A^{\succeq}\}$.

(5) $[x_j]_A^{\succeq} = [x_i]_A^{\succeq}$ iff $f(x_i, a) = f(x_j, a)$ $(\forall a \in A)$.

(6) $\mathcal{J} = \cup\{[x]_A^{\succeq} | x \in U\}$ constitute a covering of U.

For any subset X of U, and A of $\mathcal{I}^{\succeq}$ define

$$
\begin{aligned}
\underline{R_A^{\succeq}}(X) &= \{x \in U | [x]_A^{\succeq} \subseteq X\}; \\
\overline{R_A^{\succeq}}(X) &= \{x \in U | [x]_A^{\succeq} \cap X \neq \phi\}.
\end{aligned}
$$

$\underline{R_A^{\succeq}}(X)$ and $\overline{R_A^{\succeq}}(x)$ are said to be the lower and upper approximation of X with respect to a dominance relation $R_A^{\succeq}$. And the approximations have also some properties which are similar to those of Pawlak approximation spaces.

Definition 2.2(See [4])   For a ordered information system with decisions $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$, if $R_A^{\succeq} \subseteq R_D^{\succeq}$, then this information system is consistent, otherwise, this information system is inconsistent.

For simple description, the following information system with decisions are based on dominance relations, i.e. ordered information systems.

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, and denote

$$D_k \in U/R_D^{\succeq}, \quad (k \in \{1, 2, \cdots, r\}),$$
$$\underline{\eta_B} = (\underline{R_B^{\succeq}}(D_1), \underline{R_B^{\succeq}}(D_2), \cdots, \underline{R_B^{\succeq}}(D_r)),$$

the following definition is gave.

Definition 2.3(See [5]) If $\underline{\eta_B} = \underline{\eta_A}$, for all $B \subseteq A$, we say that B is a lower approximation consistent set of $\mathcal{I}^{\succeq}$. If B is a lower approximation consistent set, and no proper subset of B is lower approximation consistent set, then B is called a lower approximation reduction of $\mathcal{I}^{\succeq}$.

From the above, we can find that a lower approximation consistent set preserves the lower approximation of every decision class.

Theorem 2.1(See [5]) Let $\mathcal{I}^{\succeq}=(U, A \cup D, F, G)$ be an inconsistent ordered information system, $B \subseteq A$, then B is a lower approximation consistent set if and only if there exist $b \in B$ such that $f_b(x) > f_b(y)$ when $x \in \underline{R_A^{\succeq}}(D_k)$ and $y \notin \underline{R_A^{\succeq}}(D_k)$ for every $D_k \in U/R_D^{\succeq}(k = 1, 2, \cdots, r)$.

From the theorem, authors have provided approach to lower approximation reduction in inconsistent ordered systems based on indiscernablity matrices in [5]. We can find that it is not convenient to use the method by computers. So we will proposed the approach to lower approximation reduction operated simply by computers.

III. Algorithm of Matrix Computation for Lower Approximation Reduction

A. Dominance Matrices and Upper Approximation Decision Matrices

In this section, the dominance matrices and lower approximation decision matrices are proposed, and some properties are obtained.

Definition 3.1   Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, and denote

$$M_B = (m_{ij})_{n \times n},$$

where

$$
m_{ij} = \begin{cases} 1, & x_j \in [x_i]_B^{\succeq}, \\ 0, & \text{otherwise.} \end{cases}
$$

The matrix M_B is called dominance matrix of attributes set $B \subseteq A$. If $|B| = l$, we say that the order of M_B is l.

Definition 3.2   Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, and dominance matrices M_B, M_C of attributes sets $B, C \subseteq A$. The intersection of M_B and M_C is defined by

$$
\begin{aligned}
M_B \cap M_C &= (m_{ij})_{n \times n} \cap (m'_{ij})_{n \times n} \\
&= (\min\{m_{ij}, m'_{ij}\})_{n \times n}.
\end{aligned}
$$

The following properties are obviously.

Proposition 3.1   Let M_B, M_C be dominance matrices of attributes sets $B, C \subseteq A$, the following results always hold.

(1) $m_{ii} = 1$.

(2)   If M_B, M_C, then $M_{B \cup C} = M_B \cap M_C$.

Definition 3.3   Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, and denote

$$M_D = (r_{ij})_{n \times n},$$

where

$$
r_{ij} = \begin{cases} 0, & x_i \in \underline{R_A^{\succeq}}(D_{k_0}) \text{ and } x_j \notin \underline{R_A^{\succeq}}(D_{k_0}) \\ & \text{hold at same time for some } k_0 \in \{1, 2, \cdots, r\}. \\ 1, & \text{otherwise.} \end{cases}
$$

The matrix M_D is called lower approximation decision matrix of $\mathcal{I}^{\succeq}$.

From the above, we can see that the dominance relation of objects is decided by dominance matrices, and different decisions of objects is decided by lower approximation decision matrix.

Definition 3.4 Let $\alpha = (a_1, a_2, \ldots, a_n)$ and $\beta = (b_1, b_2, \ldots, b_n)$ be two n dimension vectors. If $a_i \le b_i, (i = 1, 2, \ldots, n)$, we say vector α is less than vector β, denoted by $\alpha \le \beta$.

Definition 3.5 Let $M_A = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ and $M_B = (\beta_1, \beta_2, \ldots, \beta_n)^T$, be two matrices, α_i and β_i be row vectors respectively. If $\alpha_i \le \beta_i$, we say M_A is less than M_B, denoted by $M_A \le M_B$.

By the definitions, dominance matrices have the following properties straightly.

Proposition 3.2 Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, and $B \subseteq A$. If M_A and M_B are the dominance matrices, then $M_A \le M_B$.

B. Theories of Matrix Computation for lower approximation reduction

In the following, we will give the theory of matrix computation for lower approximation reduction in ordered information systems.

Theorem 3.1 Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, $B \subseteq A$, then B is a lower approximation consistent set if and only if $M_B \le M_D$.

Proof. "$\Rightarrow$" We need prove that if $\eta_B^{\succeq} = \eta_A^{\succeq}$ holds for $B \subseteq A$ then $M_B \le M_D$. So we only prove $r_{ij} = 1$, when $m_{ij} = 1$. In fact, we can have that

$$m_{ij} = 1 \quad \Rightarrow \quad x_j \in [x_i]_B^{\succeq}$$
$$\Rightarrow \quad [x_j]_B^{\succeq} \subseteq [x_i]_B^{\succeq}$$

One can obtain that if $x_i \in \underline{R_A^{\succeq}}(D_{k_0})$ then $x_j \in \underline{R_A^{\succeq}}(D_{k_0})$ for $k_0 \in \{1, 2, \cdots, r\}$. That is to say $x_i \in \underline{R_A^{\succeq}}(D_{k_0})$ and $x_j \notin \underline{R_A^{\succeq}}(D_{k_0})$ do not hold at same time.

Hence, we have $r_{ij} = 1$.

"$\Leftarrow$" Suppose B be not a lower approximation consistent set, then there must exist some $D_{k_0} \in U/R_D^{\succeq}$, $(k_0 \in \{1, 2, \cdots, r\})$ such that $\underline{R_A^{\succeq}}(D_{k_0}) \ne \underline{R_B^{\succeq}}(D_{k_0})$. That is to say that $x_i \in \underline{R_A^{\succeq}}(D_{k_0})$, but $x_i \notin \underline{R_B^{\succeq}}(D_{k_0})$. So we have $[x_i]_A^{\succeq} \subseteq D_{k_0}$ and $[x_i]_B^{\succeq} \not\subseteq D_{k_0}$. By $[x_i]_A^{\succeq} \subseteq D_{k_0}$, we can have $x_i \in \underline{R_A^{\succeq}}(D_{k_0})$.

On the other hand, by $[x_i]_B^{\succeq} \not\subseteq D_{k_0}$, we know there must exist a element x_j such that $x_j \in [x_i]_B^{\succeq}$ but $x_j \notin D_{k_0}$. So $[x_j]_A^{\succeq} \not\subseteq D_{k_0}$. Thus $x_j \notin \underline{R_A^{\succeq}}(D_{k_0})$.

From above, we have got $x_i \in \underline{R_A^{\succeq}}(D_{k_0})$ and $x_j \notin \underline{R_A^{\succeq}}(D_{k_0})$, which means $r_{ij} = 0$. Since $M_B \le M_D$, so we have $m_{ij} = 0$, which is a contradiction with $x_j \in [x_i]_B^{\succeq}$.

Hence, B is a lower approximation consistent set of $\mathcal{I}^{\succeq}$ if $M_B \le M_D$.

The theorem is proved.

Corollary 3.1 Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system, and $B \subseteq A$. B is a lower

approximation reduction of $\mathcal{I}^{\succeq}$ if and only if $M_B \le M_D$ and $M_{B'} \le M_D$ does not hold for all proper subset B' of B.

C. Algorithm of Matrix Computation for lower approximation reduction

Let $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$ be an inconsistent ordered information system. We denote the dominance matrix of attributes sets B by $M_B = (\beta_1, \beta_2, \ldots, \beta_n)^T$, and lower approximation decision matrix of I by $M_D = (\gamma_1, \gamma_2, \ldots, \gamma_n)^T$, where β_i, γ_i is the ith row vectors of M_B and M_D respectively, and X^T means transposed matrix of matrix X. So we can obtain the following algorithm by Theorem 3.1 and Corollary 3.1.

Algorithm Algorithm of matrix computation for lower approximation reduction in inconsistent ordered information systems is described as follows:

Input: An inconsistent ordered information system $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$, where $U = \{x_1, x_2, \ldots, x_n\}$ and $A = \{a_1, a_2, \ldots, a_p\}$.

Output: lower approximation reductions of $\mathcal{I}^{\succeq} = (U, A \cup D, F, G)$.

Step 1 Simplify the system by combining the objects with same values of every attribute.

Step 2 Calculate lower approximation decision matrix of $\mathcal{I}^{\succeq}$: $M_D = (\gamma_1, \gamma_2, \ldots, \gamma_n)^T$.

Step 3 For all $a_l \in A, (1 \le l \le p)$, calculate the first order dominance matrices.

$$M_{\{a_l\}} = M_{\{a_l\}}^{(1)} = (\tau_1^{(1)}, \tau_2^{(1)}, \ldots, \tau_n^{(1)})^T.$$

For $i = 1$ to n.

If $0 \ne \tau_i^{(1)} \le \gamma_i$, then let $\tau_i^{(1)} = 0$,

Denote the new matrix by $FM_{\{a_l\}}^{(1)}$, and turn into next step.

Step 4 Call matrix $FM_{\{a_l\}}^{(1)} = (\tau_1^{(1)}, \tau_2^{(1)}, \ldots, \tau_n^{(1)})^T$, $a_l \in A, (1 \le l \le p)$ to be the first order upper approximation matrix. If $FM_{\{a_l\}}^{(1)} = 0$, then obtain an the first order lower approximation reduction: $\{a_l\}$. Otherwise, turn into next step.

Step 5 Calculate the intersection of all the first order nonzero matrix which are obtained in step 3, and call new matrices to be the second order dominance matrices, denoted by $M_{\{a_l a_s\}}^{(2)}$, $(M_{\{a_l a_s\}}^{(2)} \ne M_{\{a_l\}}^{(1)}, M_{\{a_l a_s\}}^{(2)} \ne M_{\{a_s\}}^{(1)})$.

Go back to step 3 and calculate all the second order lower approximation reductions.

Step 6 Obtain the higher order lower approximation reductions by repeating step 5. If the new matrices are zero matrices, then output all lower approximation reductions and terminate the algorithm.

From the above algorithm, we can know that the complication of times is $O(|U^2|2^{|A|})$ easily.

IV. AN EXAMPLE

Example Given an ordered information system in the following Table 1 and compute all lower approximation reductions. From the table, we can easily examine the system is an inconsistent ordered information system.

Table 1. An ordered information system

U	a_1	a_2	a_3	d
x_1	1	2	1	2
x_2	3	2	2	1
x_3	1	1	2	3
x_4	2	1	3	2
x_5	3	3	2	1
x_6	3	2	3	3

From the table, we have

$$[x_1]_A^{\succeq} = \{x_1, x_2, x_5, x_6\};$$
$$[x_2]_A^{\succeq} = \{x_2, x_5, x_6\};$$
$$[x_3]_A^{\succeq} = \{x_2, x_3, x_4, x_5, x_6\};$$
$$[x_4]_A^{\succeq} = \{x_4, x_6\};$$
$$[x_5]_A^{\succeq} = \{x_5\};$$
$$[x_6]_A^{\succeq} = \{x_6\};$$

and

$$D_1 = [x_1]_d^{\succeq} = [x_4]_d^{\succeq} = \{x_1, x_3, x_4, x_6\};$$
$$D_2 = [x_3]_d^{\succeq} = [x_6]_d^{\succeq} = \{x_3, x_6\};$$
$$D_3 = [x_2]_d^{\succeq} = [x_5]_d^{\succeq} = \{x_1, x_2, x_3, x_4, x_5, x_6\}.$$

Obviously, by the above, we have $R_A^{\succeq} \not\subseteq R_d^{\succeq}$, so the system in Table 1 is inconsistent.

Moreover, we can obtain

$$\underline{\eta_A} = (\underline{R_A^{\succeq}(D_1)}, \underline{R_A^{\succeq}(D_2)}, \underline{R_A^{\succeq}(D_3)})$$
$$= (\{x_4, x_6\}, \{x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}).$$

So, we can compute the dominance matrices and upper approximation decision matrices, which are

$$M_{\{a_1\}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$M_{\{a_2\}} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$M_{\{a_3\}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix};$$

$$M_D = M_{\{d\}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

By comparing matrices $M_{\{a_1\}}, M_{\{a_2\}}, M_{\{a_3\}}$ and $M_{\{d\}}$, we can find that vectors of the first, second, third, and 5th row in matrix $M_{\{a_1\}}$, $M_{\{a_2\}}$ and $M_{\{a_3\}}$ are less then those in matrix $M_{\{d\}}$ respectively, but $M_{\{a_i\}} \not\preceq M_D$, for $i = 1, 2, 3$. So the system has not the first order lower approximation reduction. Thus the first order upper approximation matrices are as follows:

$$FM_{\{a_1\}}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$FM_{\{a_2\}}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$FM_{\{a_3\}}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Furthermore, the second order upper approximation matrices are

$$M_{\{a_1 a_2\}}^{(2)} = FM_{\{a_1\}}^{(1)} \cap FM_{\{a_2\}}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix};$$

$$M_{\{a_1 a_3\}}^{(2)} = FM_{\{a_1\}}^{(1)} \cap FM_{\{a_3\}}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

$$M_{\{a_2 a_3\}}^{(2)} = FM_{\{a_2\}}^{(1)} \cap FM_{\{a_3\}}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

So, we can see that $M^{(2)}_{\{a_1a_3\}} = M^{(2)}_{\{a_2a_3\}}$, and the 6th row vectors of them are less then those of $M_{\{d\}}$ respectively, by comparing $M^{(2)}_{\{a_1a_2\}}$, $M^{(2)}_{\{a_1a_3\}}$, $M^{(2)}_{\{a_2a_3\}}$ and $M_{\{d\}}$. Hence, we can obtain all the second order lower approximation reductions, which are $\{a_1, a_3\}, \{a_2, a_3\}$.

In the next, we have

$$FM^{(2)}_{\{a_1a_3\}} = FM^{(2)}_{\{a_2a_3\}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

So, the algorithm is terminated.

Thus, all lower approximation reductions are $\{a_1, a_3\}$, $\{a_2, a_3\}$ in the system of above example.

From the example, we can find that the algorithm is valid, and operated simply, for systems with a great deal of objects and attributes.

V. CONCLUSION

It is well known that most of information systems are based on dominance relations because of various factors in practise. Therefore, it is meaningful to study the knowledge reductions in inconsistent ordered information system. In this article, the dominance matrix and lower approximation decision matrix are introduced in information systems based on dominance relations. Furthermore, the algorithm of lower approximation reduction is obtained, from which we can provide new approach to knowledge reductions in inconsistent systems based on dominance relations. Finally, an example illustrates the validity of this method, and shows the method is applicable to a complicated information system.

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